Fluctuations of the luminosity distance

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December 2005

Outline

- Definition and utility of the luminosity distance d_L
- Generic formulation of d_L in a general universe
- Calculation of d_L in a perturbed universe at first order
- Calculation of the luminosity distance power spectrum

Luminosity distance

$$d_L^2 = \frac{L}{4\pi F}$$

L	Luminosity
F	Flux

- Euclidean universe $d_L = R$
- Homogeneous and isotropic universe in expansion
  $\Rightarrow$ redshift + surface stretching
- Universe with perturbations
  $\Rightarrow$ redshift + surface deformation + deflection

Conclusion : the luminosity distance depends intimately on the structure of the universe

Utility of the luminosity distance

d_L measurable quantity : supernovae = standard candle

Homogeneous and isotropic universe : the data correspond to a universe in acceleration.

Idea

- Calculate the luminosity distance in a realistic model of universe (with perturbations)
- Calculate the power spectrum of the luminosity distance
- Compare with the measurements in order to get information about the cosmological parameters

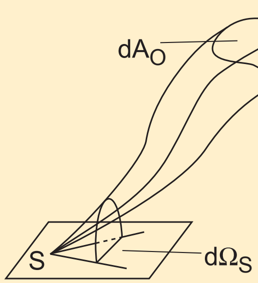
Luminosity distance

Emission $$L = \frac{n_S E_S}{\Delta\tau_S}$$

Reception $g(x_O)dA_O$: photon density on dA_O

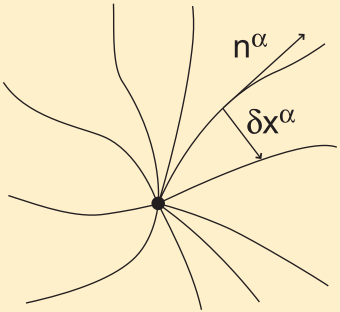
$$\Rightarrow \quad F = \frac{n_S g(x_O) E_O}{\Delta\tau_O}$$

Photon number conservation $$n_S \frac{d\Omega_S}{4\pi} = n_S g(x_O) dA_O$$



$$d_L^2 = \frac{E_S}{E_O}\frac{\Delta\tau_O}{\Delta\tau_S}\frac{dA_O}{d\Omega_S} = (z+1)^2\frac{dA_O}{d\Omega_S}$$

Reception surface and emission solid angle



family of geodesics $x^\mu(\lambda, y^1, y^2)$

Definition of two four-vectors :

$$n^\alpha = \frac{\partial x^\alpha}{\partial \lambda}$$

$$\delta x^\alpha = \sum_{i=1,2} \frac{\partial x^\alpha}{\partial y^i} \delta y^i$$

$$n^\alpha \delta x_\alpha = 0$$

Jacobi map

At the observer : δx^α describes dA_O

At the source : $\delta\theta^\alpha = \frac{1}{E_S}\nabla_n(\delta x^\alpha)$ describes $d\Omega_S$

System of coupled differential equations

$$\begin{aligned}\nabla_n(E_S\delta\theta^\alpha) &= R^\alpha_{\rho\mu\nu}n^\rho n^\mu \delta x^\nu \\ \nabla_n(\delta x^\alpha) &= E_S\delta\theta^\alpha\end{aligned}$$

$$\delta x^\alpha(\lambda_O) = J^\alpha_\beta(\lambda_O,\lambda_S)\delta\theta^\beta(\lambda_S) \qquad J : \text{Jacobi map}$$

$$\frac{dA_O}{d\Omega_S} = \det J$$

The luminosity distance

Conformal transformation doesn't change the null geodesics

$$\tilde{g}_{\mu\nu} = a^2(\eta) g_{\mu\nu} \quad \Rightarrow \quad \tilde{d}_L = \frac{a_O^2}{a_S} d_L$$

Perturbed universe, flat ($K = 0$), newtonian gauge, no anisotropic stress

$$g_{\mu\nu} dx^\mu dx^\nu = -\big(1 + 2\psi(\eta, \mathbf{x})\big) d\eta^2 + \big(1 - 2\psi(\eta, \mathbf{x})\big) \delta_{ij} dx^i dx^j$$

Redshift : $\quad z+1 = \dfrac{E_S}{E_O} = 1+\psi_O-\psi_S-2\displaystyle\int_{\lambda_S}^{\lambda_O} d\lambda \dot{\psi} \bar{n}^0 + \mathbf{v}_O \mathbf{n} - \mathbf{v}_S \mathbf{n}$

Jacobi map J : $\quad$ depends on the Riemann tensor

Luminosity distance in the expanding universe

$$
\begin{aligned}
\tilde{d}_L(z_S, \mathbf{n}) \; &= \; (1+z_S)\Bigg\{ (\eta_O - \eta_S) + \frac{1}{\mathcal{H}_S}\left(\Psi_O - \mathbf{v}_O \cdot \mathbf{n}\right) \\
&- \; \left(\eta_O - \eta_S + \mathcal{H}_S^{-1}\right)\Psi_S + 2\int_{\eta_S}^{\eta_O} d\eta \Psi(\eta) \\
&+ \; 2\mathbf{n}\cdot\Bigg[-\frac{1}{\mathcal{H}_S}\int_{\eta_S}^{\eta_O} d\eta \nabla\Psi(\eta) + \int_{\eta_S}^{\eta_O} d\eta \int_{\eta_S}^{\eta} d\eta' \nabla\Psi(\eta') \\
&\quad + \frac{\eta_O - \eta_S - \mathcal{H}_S^{-1}}{8\pi G a_S^2 (\rho+p)(\eta_S)}\left(\mathcal{H}\nabla\Psi + \nabla\dot{\Psi}\right)(\eta_S)\Bigg] \\
&- \; \int_{\eta_S}^{\eta_O} d\eta \int_{\eta_S}^{\eta} d\eta' (\eta' - \eta_S)\Big(\nabla^2\Psi(\eta') - \partial_i\partial_j\Psi(\eta') n^i n^j\Big)\Bigg\}
\end{aligned}
$$

The luminosity distance power spectrum

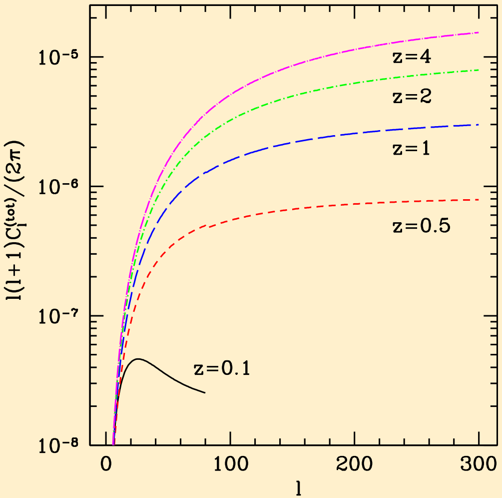
$$d_L(z_S, \mathbf{n}) = \sum_{lm} a_{lm}(z_S) Y_{lm}(\mathbf{n})$$

$a_{lm}(z_S)$ statistical variable $\qquad C_l(z_S, z_{S'}) = \dfrac{\langle a_{lm}(z_S) a^{\star}_{lm}(z_{S'}) \rangle}{\bar{d}_L(z_S)\bar{d}_L(z_{S'})}$

$$\frac{\langle d_L(z_S, \mathbf{n}) d_L(z_{S'}, \mathbf{n}') \rangle}{\bar{d}_L(z_S)\bar{d}_L(z_{S'})} = \sum_l \frac{2l+1}{4\pi} C_l(z_S, z_{S'}) P_l(\mathbf{n}\mathbf{n}')$$

C_l is a measurable quantity $\Rightarrow$ we can constrain the cosmological parameters in comparing the data with the calculation

Results for a pure CDM universe



- 10^{-5} instead of $\langle \Psi^2 \rangle \sim 10^{-10}$
- Effect of the deflections due to the gravitational potential $\nabla^2 \Psi \sim \frac{\delta\rho}{\rho}$

$$C_l \sim \frac{1}{(\eta_O - \eta_S)(\eta_O - \eta_{S'})} \int_{\eta_S}^{\eta_O} d\eta \int_{\eta_S}^{\eta} d\eta' (\eta' - \eta_S)$$
$$\int_{\eta_{S'}}^{\eta_O} d\tilde{\eta} \int_{\eta_{S'}}^{\tilde{\eta}} d\tilde{\eta}' (\tilde{\eta}' - \eta_{S'}) \langle \nabla^2 \Psi(\eta', \mathbf{x}) \nabla^2 \Psi(\tilde{\eta}', \mathbf{x}') \rangle$$

Results for a pure CDM universe

Impact of the deflections

- Small deflection $\Rightarrow$ large effect at large distance
- Integrated effect : after N fluctuations $\mapsto \sqrt{N}\cdot$deflection

Which scale contribute the most ?

The largest contribution comes from the scale which enters horizon at equality

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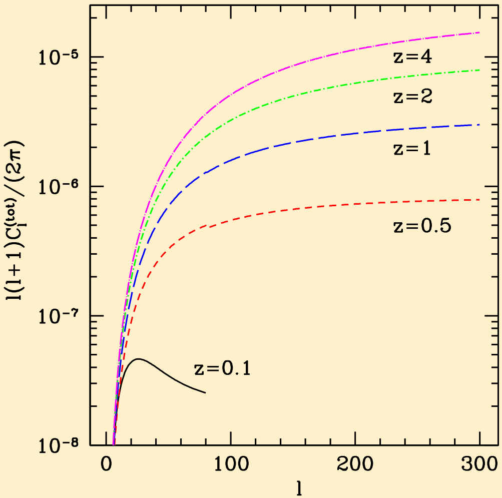

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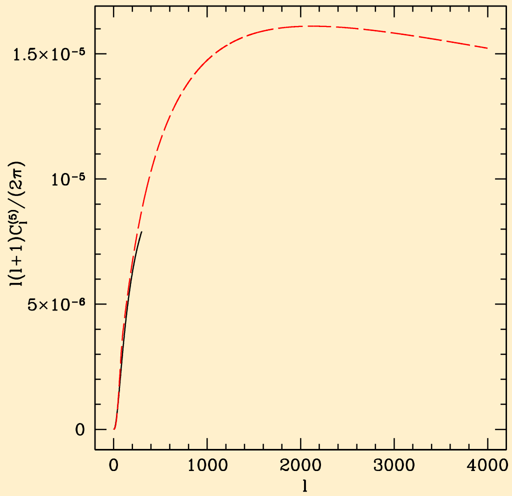

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Impact on parameters

- Need of dark energy ?

variance $$\frac{\langle d_L^2(z_S, \mathbf{n})\rangle}{\bar{d}_L^2(z_S)} = \sum_l \frac{2l+1}{4\pi} C_l(z_S) \sim 10^{-5}$$

- New tool $C_l(z_S)$ to calculate the cosmological parameters
  - Numerical calculation of $\mathbf{C_l}$ for a universe with **dark energy**
  - Calculation of the **dipole** : velocity known from CMB dipole, $C_1(z_S) \sim \frac{1}{\mathcal{H}^2(z_S)}$
  - **Observational** point of view : numerous supernovae (ALPACA)

Conclusion

- Until now (astro-ph/05 11 183)
  - General formulation of the luminosity distance
  - Application in a perturbed universe at first order
  - Calculation of the correlation coefficient in a CDM universe

- Future
  - Near future : calculation of the dipole in a more realistic universe
  - Numerical calculation of the other multipoles

The dipole in a CDM universe