Transient accelerated expansion and double quintessence

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Presentation

- **Introduction**
- **Pure exponential potential**
- **“ Assisted Quintessence ”**
- **Albrecht & Skordis potential**
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D. Blais and D. Polarski, PRD 70, 084008 (2004)

Introduction

Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_p^2}(\rho_r + \rho_m + \rho_\Lambda) - \frac{k}{a^2} \ ,$$

$$\Rightarrow \left(\frac{H}{H_0}\right)^2 = \Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\Lambda,0} + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2 \ .$$

Observations

(1) magnitude-redshift relation of SN-Ia

$$\mu = m - M = 5 \log \frac{d_L}{\mathrm{Mpc}} + 25 \ ,$$

luminosity distance at a given redshift z

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = (1+z)H_0^{-1}\int_0^z \frac{H_0}{H(z')}dz' \ ,$$

joint probability distribution well approximated by

$$0.8\ \Omega_{m,0} - 0.6\ \Omega_{\Lambda,0} \sim -0.2 \pm 0.1 \ .$$

(2) CMB : first Doppler peak position

$$\Rightarrow \quad \Omega_{tot,0} = \Omega_{m,0} + \Omega_{\Lambda,0} \sim 1 \quad (\Omega_{r,0} \ll 1) \quad \Rightarrow \quad \Omega_{k,0} \sim 0 \ .$$

$\Rightarrow$ **cosmic complementarity :**

$$\boxed{(1) + (2) \Rightarrow \quad \Omega_{m,0} \sim 0.3 \ , \quad \Omega_{\Lambda,0} \sim 0.7}$$

Introduction

Observations

(3) HST key project

$$H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1} \ ,$$

(4) LSS data (2dfGRS $\Rightarrow \Omega_{m,0}h = 0.20 \pm 0.03$)

$$\Omega_{m,0} \sim 0.3 \ ,$$

$\Rightarrow$ **cosmic concordance model :**

$$\boxed{\Omega_{m,0} \sim 0.3 \ , \quad \Omega_{\Lambda,0} \sim 0.7 \ , \quad \Omega_{r,0} \sim 10^{-4} \ , \quad h \sim 0.7}$$

Deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2}(1 + \Omega_r - 3\ \Omega_\Lambda) \ ,$$

$$\Rightarrow q_0 \simeq \frac{1}{2}(1 - 3\ \Omega_{\Lambda,0}) \simeq -0.55 \quad (\Omega_{r,0} \ll 1) \ ,$$

$$\boxed{\text{universe presently accelerating : } q_0 < 0}$$

Perfect fluid with density ρ_{DE} and pression p_{DE} ($p_{DE} = w_{DE}\rho_{DE}$)

$$\Rightarrow q = \frac{1}{2}(1 + \Omega_r + 3\ w_{DE}\Omega_{DE}) \ ,$$

Accelerated expansion if $w_{DE}\Omega_{DE} < -\frac{1}{3}$.

Introduction

(1) Old cosmological constant problem

Why the value is so small compared to QFT calculations ?

$$\rho_{\Lambda,0} = \frac{\Lambda c^2}{8\pi G} \sim 10^{-123}\ m_p^4\ .$$

Or why all contributions cancel each other up to a fantastic precision ?

$$T^{\Phi}_{\mu\nu} = V(\Phi_{min}) g_{\mu\nu} \Rightarrow \Lambda_{eff} = \Lambda + 8\pi G V(\Phi_{min})\ .$$

(2) New cosmological constant problem

Why is the acceleration happening during the present epoch of the cosmic evolution ? ($q(z \lesssim 0.7) < 0$)

Cosmic coincidence : why $\Omega_{\Lambda,0} \sim \Omega_{m,0}$?

(3) Eternal accelerated expansion ?

Troublesome from the standpoint of string theory since the de Sitter horizon does not allow one to define the S-matrix. Asymptotic states are inconsistent with spacetimes that exhibit event horizons.

However, <u>eternal acceleration for most of Dark Energy models !</u>

Introduction

(4) What is causing the cosmic acceleration ?

Cosmological constant Λ :

Perfect fluid with $p_\Lambda = -\rho_\Lambda$, i.e. $w_\Lambda = -1$

fine-tuning and cosmic coincidence problem not solved.

Kinematic & hydrodynamic models :

$$\rho_{DE} = f(a) \quad \{= a^{-\alpha}\} \ ,$$

equivalent to

$$p_{DE} \equiv w_{DE}\rho_{DE} = -\rho_{DE}\left(1 + \frac{1}{3}\frac{\mathrm{d}\ln\rho_{DE}}{\mathrm{d}\ln a}\right) \quad \{= \left(\frac{\alpha}{3} - 1\right)\rho_{DE}\} \ .$$

Quintessence models :

Slow-rolling scalar field Φ along a nearly flat potential V

$$\text{KG} \quad : \quad \ddot{\Phi} + 3H\dot{\Phi} + \frac{\partial V}{\partial \Phi} = 0 \ ,$$

$$\text{eos} \quad : \quad w_\Phi \equiv \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{1}{2}\dot{\Phi}^2 - V(\Phi)}{\frac{1}{2}\dot{\Phi}^2 + V(\Phi)} \ ,$$

if $\dot{\Phi}^2 \ll V \Rightarrow w_\Phi \sim -1$
but fine-tuning $m_\Phi \equiv \sqrt{\frac{\partial^2 V}{\partial \Phi^2}} \sim H_0 \sim 10^{-33}$ eV
and cosmic coincidence problem only alleviated by the so-called tracking potential.

Pur exponential

$$V(\Phi) = M^4 \exp(-\lambda\Phi)$$

Stable solutions ($X \equiv \frac{\dot{\Phi}}{\sqrt{6}H}$, $X_V \equiv \frac{\sqrt{V}}{\sqrt{3}H}$) :

1) scaling solution if $\lambda^2 > 3(1 + w_b)$:

$$X = \sqrt{\frac{3}{2}}\frac{1 + w_b}{\lambda}\ ,\ X_V = \left[\frac{3(1 - w_b^2)}{2\lambda^2}\right]^{1/2}\quad ,$$

$$\Rightarrow \quad \boxed{\Omega_Q = \frac{3(1+w_b)}{\lambda^2}\ ,\ w_Q = w_b}$$

2) scalar field dominated solution if $\lambda^2 < 3(1 + w_b)$:

$$X = \frac{\lambda}{\sqrt{6}}\ ,\ X_V = \left[1 - \frac{\lambda^2}{6}\right]^{1/2}\quad ,$$

$$\Rightarrow \quad \boxed{\Omega_Q = 1\ ,\ w_Q = \frac{\lambda^2}{3} - 1}$$

If stable solution reached in the past,
BBN constraint : $\Omega_Q(1\text{ MeV}) \lesssim 0.045 \Rightarrow \lambda \gtrsim 10$
$\Rightarrow$ quintessence can not dominate today and
moreover $w_Q \simeq w_b = 0$ preventing acceleration.

But if the field is frozen until today ($w_Q \sim -1$) with $M^4 \sim m_p^2 H_0^2$ ("thawing regime"), model is viable with

- transient acceleration if $\sqrt{2} < \lambda \lesssim 2$
- eternal acceleration if $0 < \lambda < \sqrt{2}$

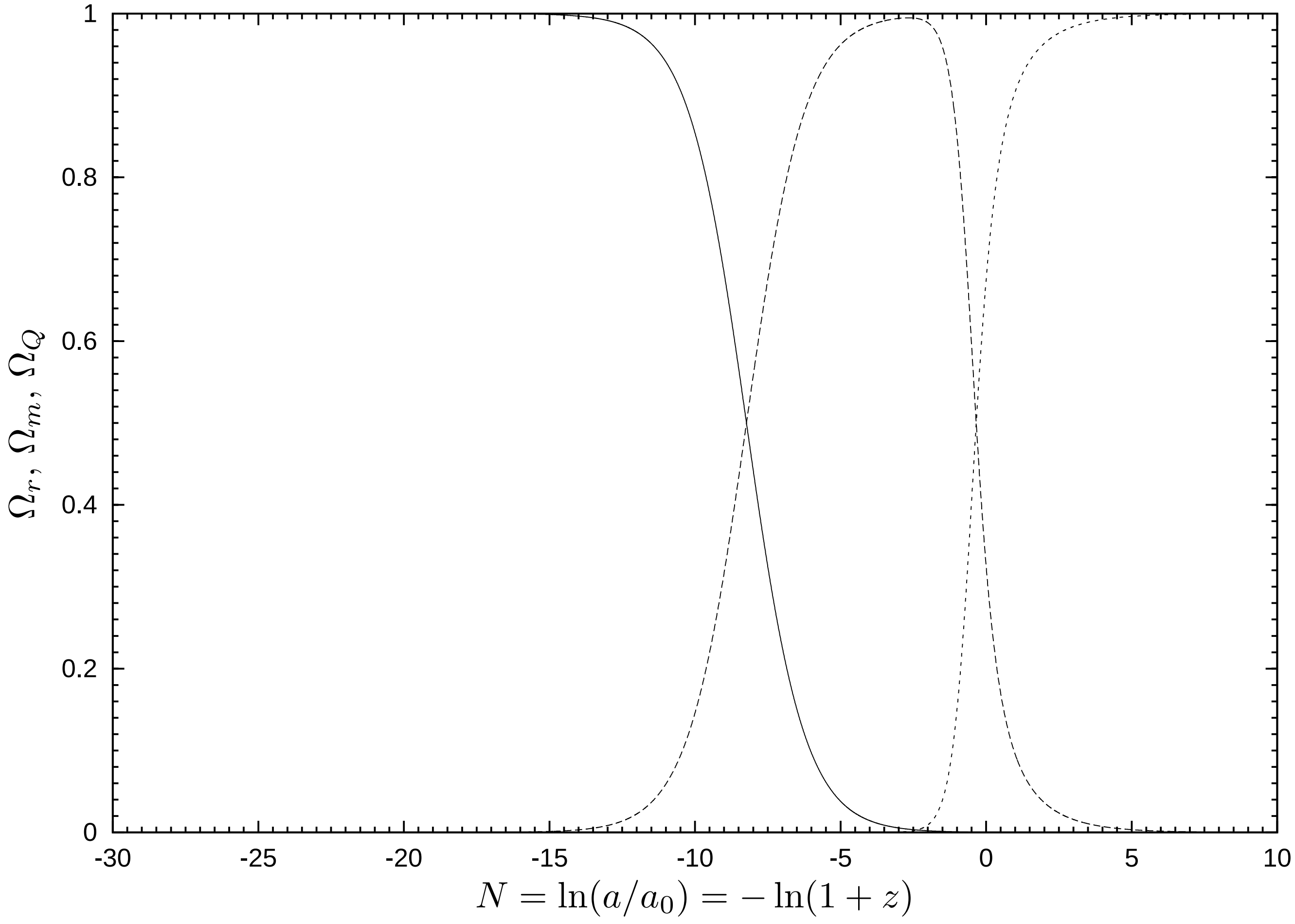

Figure 1: The evolution of the densities Ω_r (solid line), Ω_m (dashed line) and Ω_Q (dotted line) are shown with $\lambda = 1.5$.

$\Omega_{Q,0} \simeq 0.67$ and $\Omega_Q(a \to \infty) = 1$.

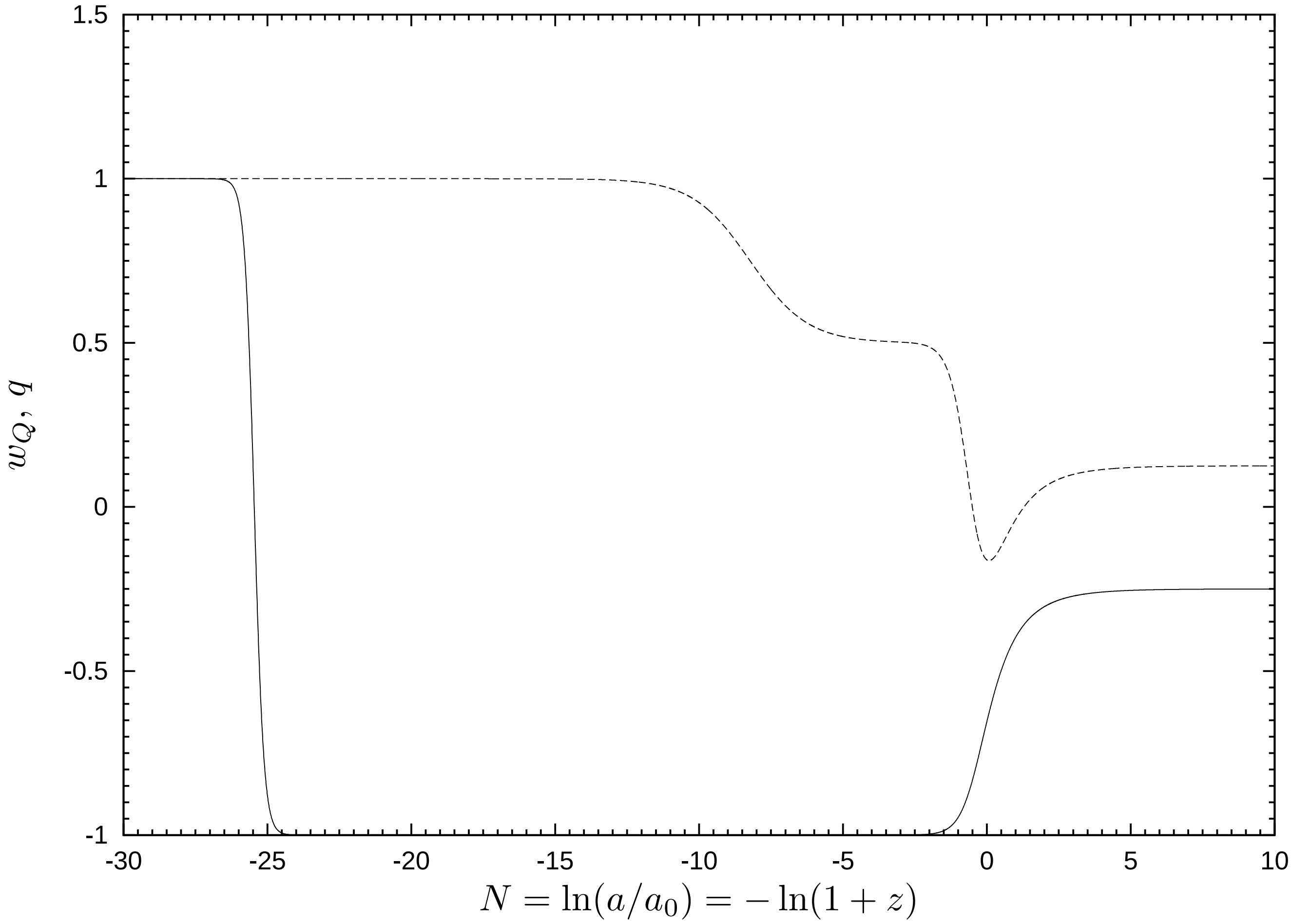

Figure 2: The evolution of w_Q (solid line) and the deceleration parameter q (dashed line) are shown for $\lambda = 1.5$.

$w_{eff} \simeq -0.78$, $w_{Q,0} \simeq -0.66$, $q_0 \simeq -0.16$ and $w_Q(a \to \infty) = -0.25$.

Assisted Quintessence

D. Blais and D. Polarski, PRD 70, 084008 (2004)
S.A. Kim, A.R. Liddle and S. Tsujikawa, PRD 72, 043506 (2005)

$$V(\Phi_i) = \sum_{i=1}^{n} M_i^4 e^{-\lambda_i \Phi_i}$$

Assisted behaviour in the presence of a perfect fluid $p = w_b \rho$

$$a(t) \propto t^n \quad \text{with} \quad n = \frac{2}{\lambda_{eff}^2},$$

where

$$\frac{1}{\lambda_{eff}^2} = \sum_{i=1}^{n} \frac{1}{\lambda_i^2} \ .$$

Expansion rate is more rapid the more fields there are.

Late-time attractors :

1) assisted scaling solution if $\lambda_{eff}^2 > 3(1 + w_b)$:

$$\Omega_Q = \frac{3(1+w_b)}{\lambda_{eff}^2} \ , \ w_Q = w_b$$

2) assisted scalar field dominated solution if $\lambda_{eff}^2 < 3(1 + w_b)$:

$$\Omega_Q = 1 \ , \ w_Q = \frac{\lambda_{eff}^2}{3} - 1$$

As the universe evolves, more and more fields would join the assisted quintessence attractor, reducing λ_{eff}. This could switch the attractor from the scaling regime into the late-time scalar field dominance.

Albrecht & Skordis potential

A. Albrecht and C. Skordis, PRL 84, 2076 (2000)

Deformation of a pur exponential to create a minimum for Φ

$$\boxed{V(\Phi) = M^4 e^{-\lambda\Phi}\ [P_0 + (\Phi - \Phi_c)^2]}$$

Minimum :

$$\Phi_\pm = \Phi_c + \frac{1}{\lambda}\left(1 \pm \sqrt{1 - \lambda^2 P_0}\right) \ .$$

Φ reaches his minimum $\Rightarrow \dot{\Phi}^2 \ll V \Rightarrow w_\Phi \simeq -1$.

Conditions on parameters :

- BBN constraint $\Omega_Q(1\ \text{MeV}) \lesssim 0.045 \Rightarrow \lambda \gtrsim 10$,
- fine-tuning on Φ_c (cosmic coincidence),
- transient acceleration if $\lambda^2 P_0 \sim 1$,
  eternal acceleration if $\lambda^2 P_0 \lesssim 1$,
  excluded by observation if $\lambda^2 P_0 \gtrsim 1$.

Most of the parameter space leads to an eternal acceleration.
All parameters (λ, M, P_0, Φ_c) take natural value in Planck units but the cosmic coincidence problem is not solved.

Potentials with coupled scalar fields

(1) most likely that an ensemble of scalar fields will emerge (moduli, axions, etc)

(2) invoked for various desirable features they exhibit (hybrid inflationary models, reheating models, etc)

(3) theories with a stable SUSY vacuum can't relax into a zero-energy ground state if acceleration is guided by one scalar field

Scalar fields with potentials of the type

$$V(\Phi, \Psi) = e^{-\lambda\Phi}\ P(\Phi, \Psi)\ ,$$

where P contains polynomial as well as interacting term in Φ and Ψ can arise in the low-energy limit of fundamental particle physics theories such as string/M-theory, phenomenological brane-world constructions.

Generalized AS potential

D. Blais and D. Polarski, PRD 70, 084008 (2004)

A very simple possibility can be proposed

$$V(\Phi,\Psi) = M^4 e^{-\lambda\Phi}\ [P_0 + \Psi^2(\Phi - \Phi_c)^2]$$

Minimum :

$$\Phi_\pm = \Phi_c + \frac{1}{\lambda}\left(1 \pm \sqrt{1 - \frac{\lambda^2 P_0}{\Psi^2}}\right) \ .$$

AS potential minimum is fixed but here minimum is dynamical and disappears if $\Psi < \Psi_c \equiv \lambda\sqrt{P_0}$

- BBN constraint $\Rightarrow \lambda \gtrsim 10$,
- fine-tuning on Φ_c (cosmic coincidence),
- acceleration is necessarily transient if $\Psi_i \gtrsim \Psi_c$, excluded by observation if $\Psi_i \lesssim \Psi_c$.

All parameters (λ, M, P_0, Φ_c) take natural value in Planck units.

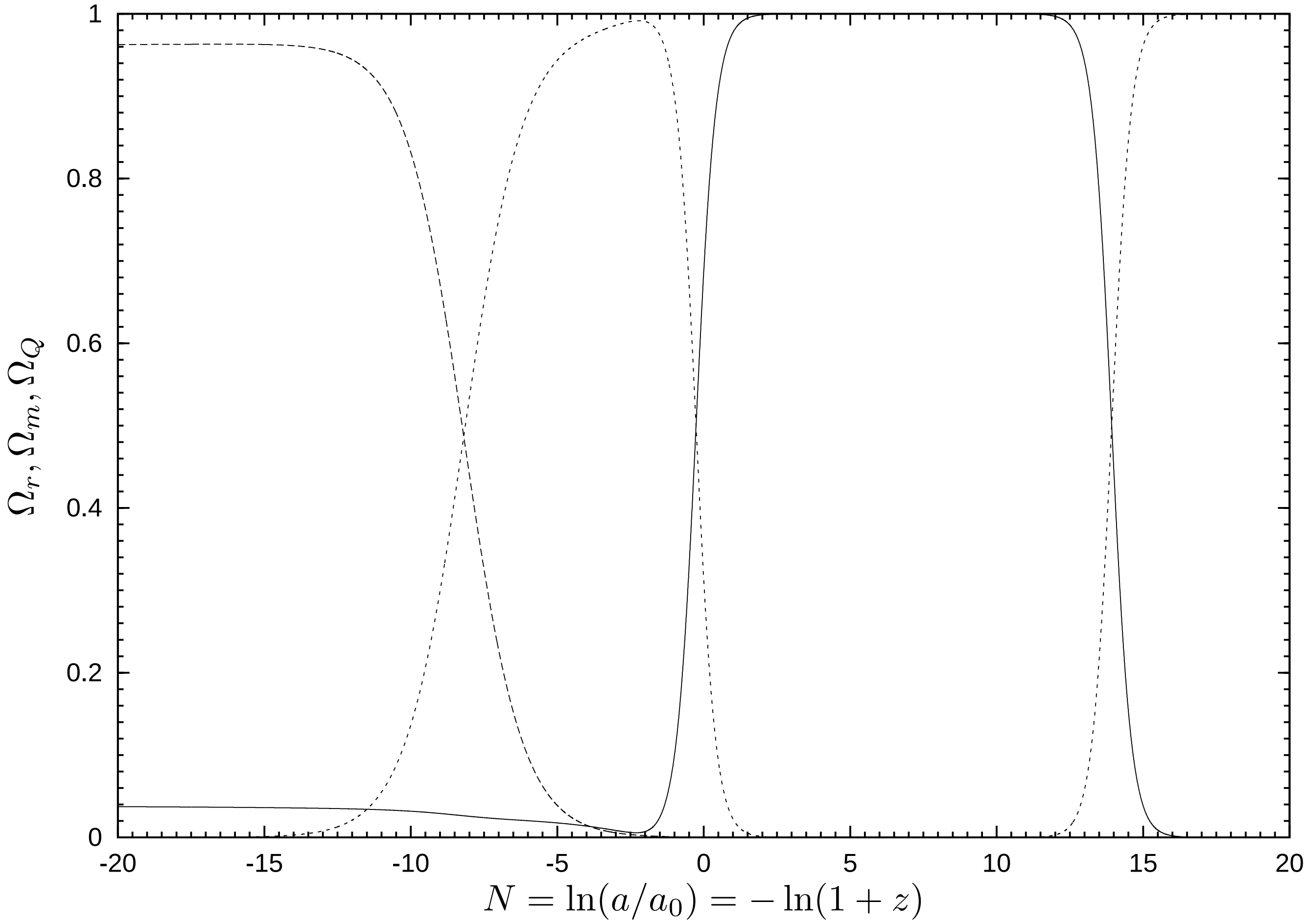

Figure 3: The evolution of the densities Ω_r (dashed line), Ω_m (dotted line) and Ω_Q (solid line) are shown with $\lambda = 10$, $P_0 = 0.1$ and $\Phi_c = 23.78$.

$\Omega_{Q,0} \simeq 0.68$, $H_0 t_0 \simeq 0.95$ and $\Omega_Q(a \to \infty) = 0.03$.

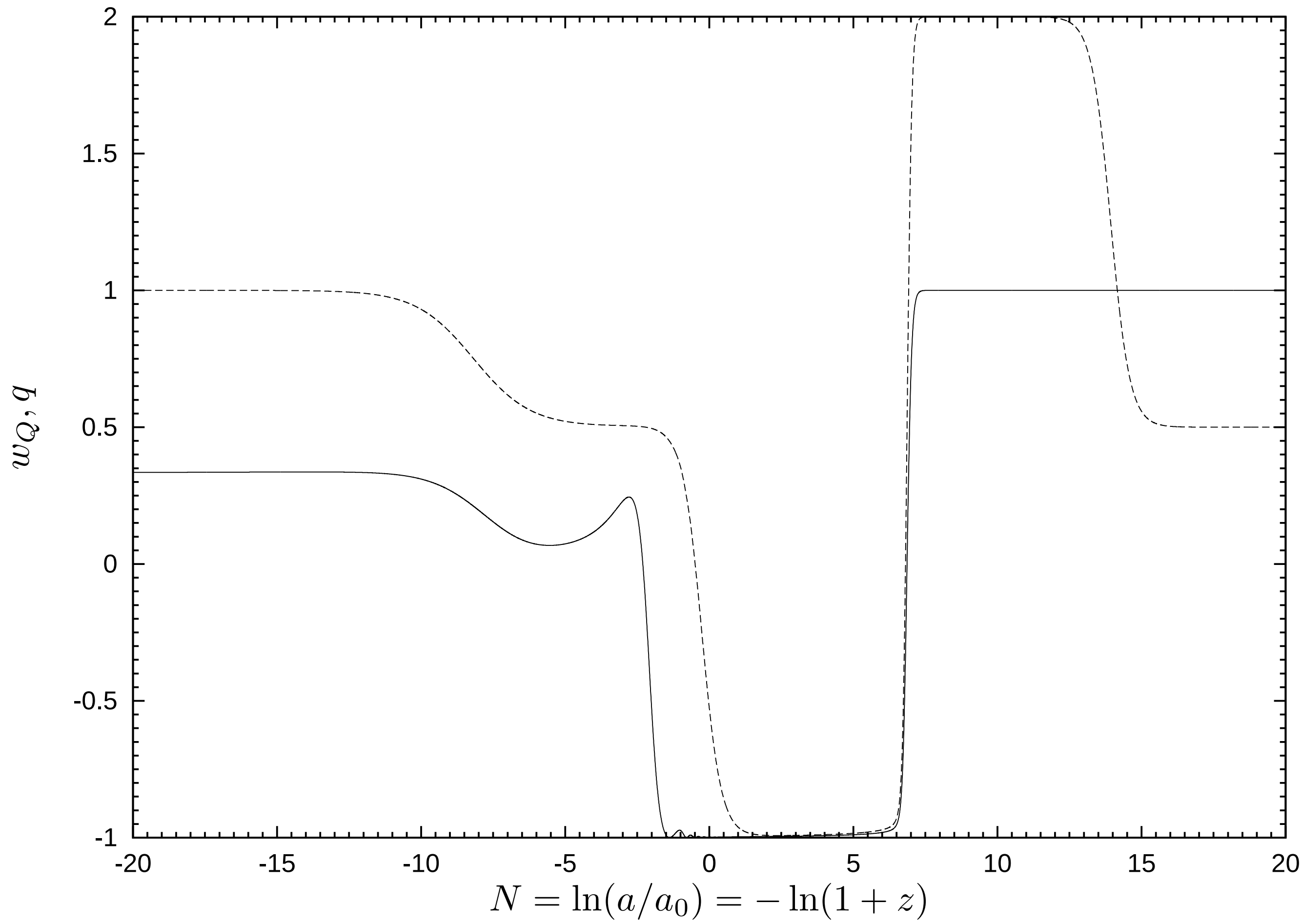

Figure 4: The evolution of w_Q (solid line) and the deceleration parameter q (dashed line) are shown with $\lambda = 10$, $P_0 = 0.1$ and $\Phi_c = 23.78$.

$w_{eff} \simeq -0.99$, $w_{Q,0} \simeq -1.0$, $q_0 \simeq -0.52$ and $w_Q(a \to \infty) = 0$.

Acceleration ended by today ?

Model for which acceleration is ended by today $q_0 > 0$!

Generalized AS model characterized by the following parameters

$$\lambda = 10 \ , \quad \Phi_c = 23.8 \ , \quad P_0 = 0.164 \quad (\Phi_i = 0 \ , \quad \Psi_i = 5) \ .$$

For this model, cosmological parameters are :

$$\Omega_{Q,0} \simeq 0.66 \ , \quad w_{eff} \simeq -0.87 \ , \quad w_{Q,0} \simeq -0.49 \ , \\ H_0 t_0 \simeq 0.91 \ , \quad d_A \simeq 8.976 \ h^{-1} \ .$$

where

$$w_{eff} \equiv \frac{\int_0^{a_0} w_Q \ \Omega_Q \ da}{\int_0^{a_0} \Omega_Q \ da}$$ G. Huey et al., PRD 59, 063005 (1999)

Decelerated expansion today : $q_0 \simeq 0.013 > 0$.

A. G. Riess et al. (SST), AJ 607, 665 (2004)
SN-Ia (ΛCDM) $\Rightarrow$ $0.26 \lesssim \Omega_{m,0} \lesssim 0.34$ at 1σ.

D.N. Spergel et al. (WMAP), APJS 148, 175 (2003)
WMAP+ACBAR+CBI+2dfGRS+Lyman α forest data (ΛCDM)
$\Rightarrow$ $h = 0.71^{+0.04}_{-0.03}$, $\Omega_{m,0} h^2 = 0.135^{+0.008}_{-0.009}$,
$d_A = 14^{+0.2}_{-0.3}$ Gpc and $t_0 = 13.7^{+0.2}_{-0.2}$ Gyrs at 1σ.

If $0.63 \leq h \leq 0.66$ $\Rightarrow$ $0.135 \leq \Omega_{m,0} h^2 \leq 0.145$,
$13.6 \leq d_A \leq 14.25$ Gpc and $13.51 \leq t_0 \leq 14.11$ Gyrs
in agreement at 2σ with SN-Ia data, WMAP data and with angular size distance d_A.

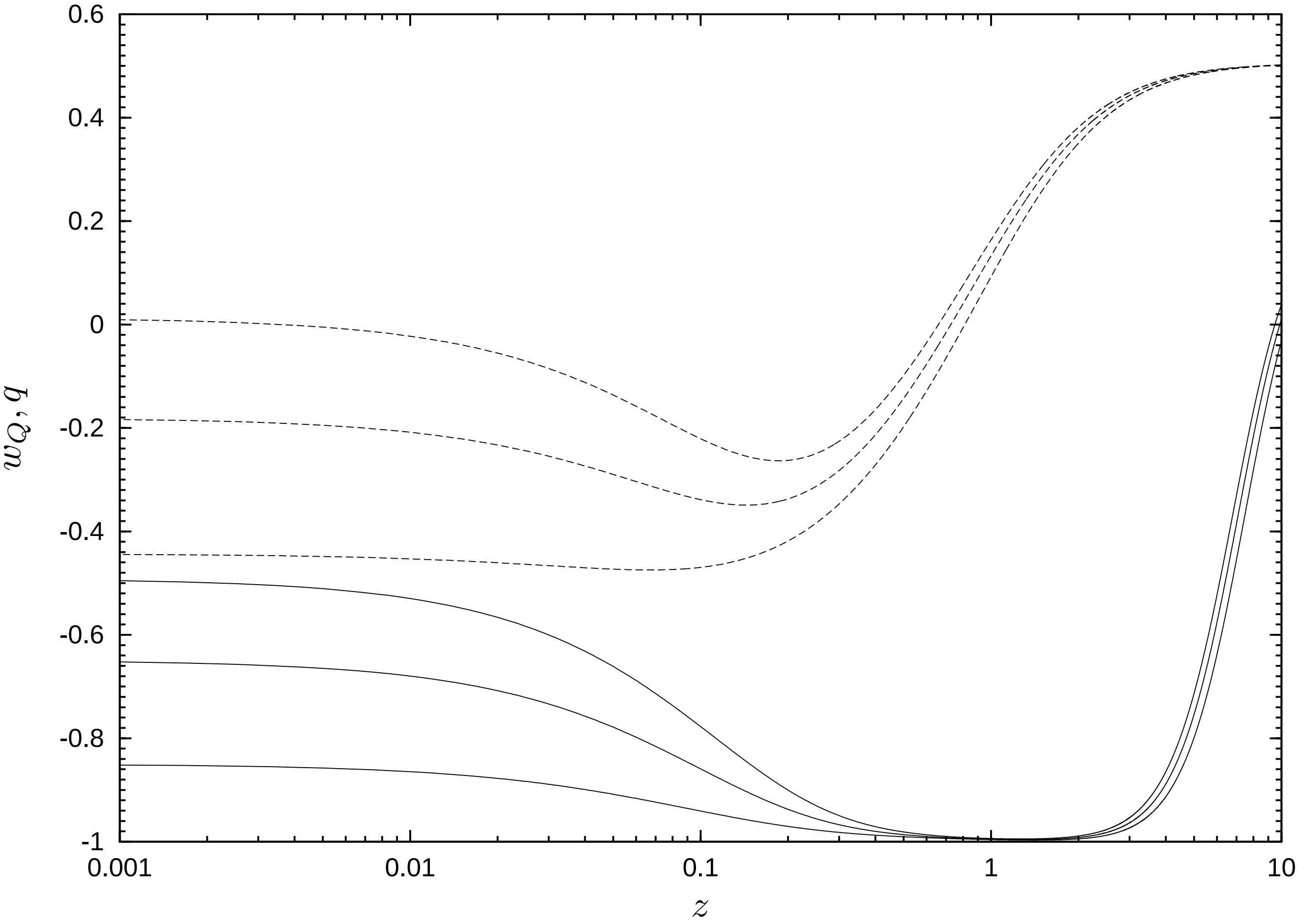

Figure 5: The evolution of the quantities w_Q (solid line) and q (dashed line) are shown versus redshift with the fixed parameters $\lambda = 10$, $\Phi_c = 23.8$ while P_0 takes the values, from bottom to top, $0.160, 0.162, 0.164$ ($\Phi_i = 0$ and $\Psi_i = 5$).

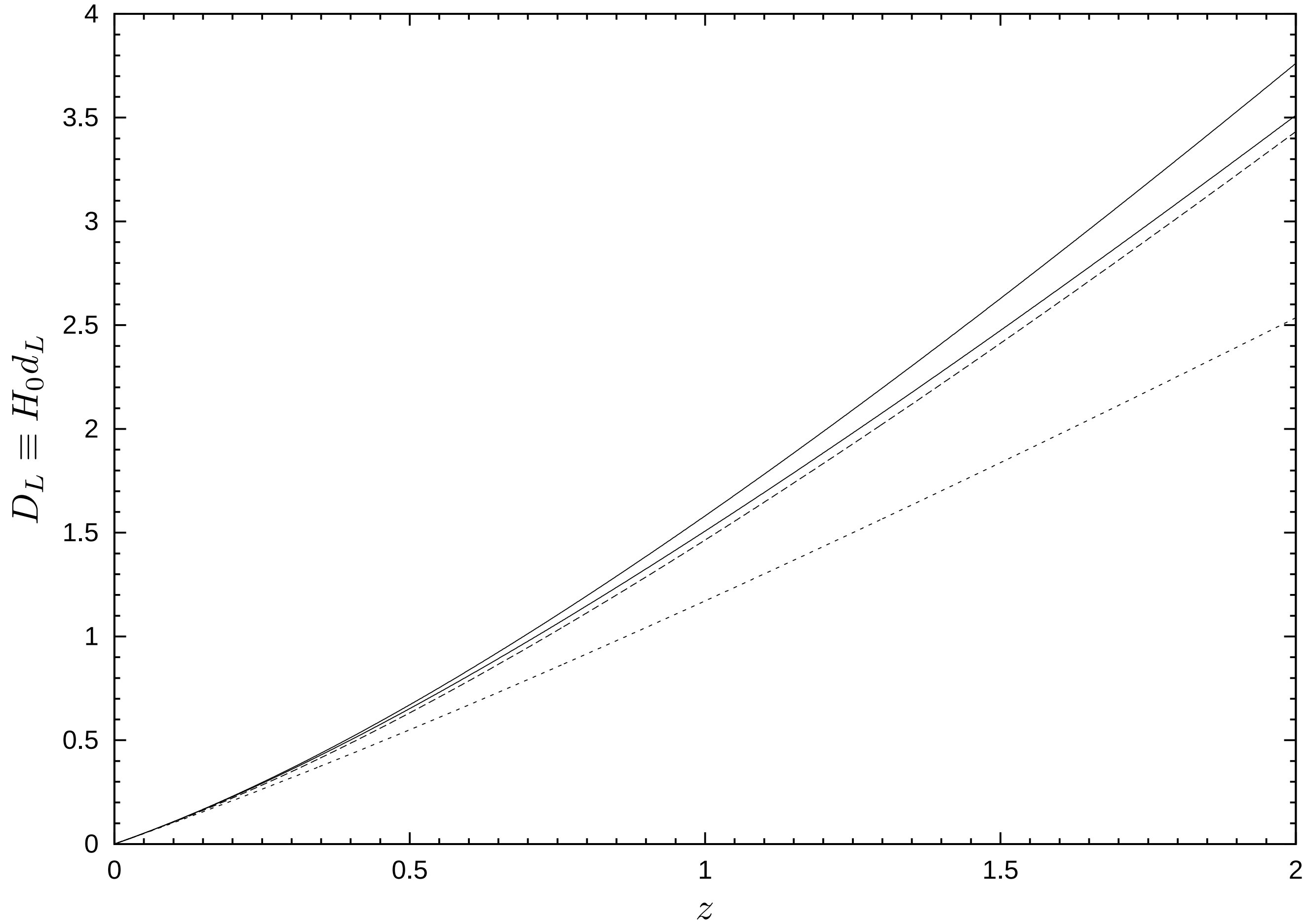

Figure 6: The following models are plotted: the SNIa data with 1σ errors corresponding to flat Λ-models with $\Omega_{\Lambda,0} = 0.74$ (upper solid line), $\Omega_{\Lambda,0} = 0.66$ (lower solid line), the Einstein-de Sitter universe (dotted line), the generalized AS potential for $P_0 = 0.164$ and accelerated expansion ended by today (dashed line).