The Hitchiker's Guide to Dark Energy

or how to get a R.I.D.E !!!

Rigidity Introduced in Dark Energy

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CONTENTS

- Why get a ride ?
- What kind of rides are available?
- Is the ride safe ?
- Where do we go from there?

B.C.C.M Phys.Rev. D72 (2005) 023503

Episode I

Things to remember from yesterday

Different ways to model DE

Modify Gravity   Cosmological Constant   Another Matter Sector

Observational facts

$w < -1/3$ is the only we can claim to be evidence

Motivations for a Solid Dark Energy

To account for DE we need $w = P/\rho < 0$

Consider a Polytropic EOS $\rho = n^{\gamma}$

For a perfect fluid $c_s^2 = \frac{dP}{d\rho} = \frac{\beta}{\rho + P}$

$$\beta = n\frac{dP}{dn} = w\gamma < 0$$

UNSTABLE !!

Motivations for a Solid Dark Energy

Isotropic Solid

$$c_L^2 = \frac{\beta + 4/3\mu}{\rho + P}$$

$$\beta = n\frac{dP}{dn}$$

$$c_T^2 = \frac{\mu}{\rho + P}$$

Stability requires

$$\mu \geq -\frac{3}{4}\beta$$

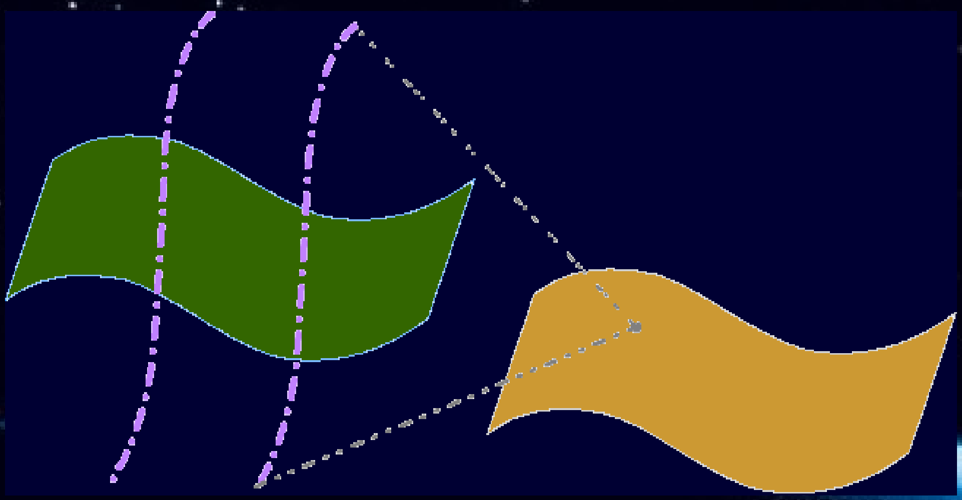
Bucher & Spergel PRD 60(1999) 043505

A Bit Of Theory

Variational Principle

$$\rho = \check{\rho} + \frac{1}{2}\check{\Sigma}^{abcd} s_{ab} s_{cd}$$

$$s_{ab} = \frac{1}{2}(\gamma_{ab} - \check{\gamma}_{ab})$$



Isotroic	i=1
Cubic	i=2
Hexagonal	i=3
Tetragonal	i=4

The weaker the symmetry the more moduli

Carter,Chachoua,Chamel
GRG 38 (Jan 2006)

The Stablility Issue

Characteristic Equation

$$\left(v^2(\check{\rho}+\check{P})\check{\gamma}^{-1\,ab} - Q^{ab}\right)\iota_a = 0$$

$$Q^{ab} = \check{\Sigma}^{acbd}\nu_c\nu_d + \check{\beta}\nu^a\nu^b$$

Cubic Symmetry

- Along the axes

$$\check{\mu}_{\mathrm{L}} = -\frac{D}{2(D-1)}\check{\beta}$$

- Away from the axes

$$\check{\mu}_{\mathrm{T}} = -\frac{D}{2(D-1)}\check{\beta}$$

Isotropic Symmetry

$$\check{\mu} = -\frac{D}{2(D-1)}\check{\beta}$$

Application to Cosmology

Nambu-Goto Topological defects

$$\gamma = \frac{(D-p)}{D}$$

$$w = -\frac{p}{D}$$

$$\breve{\beta} = -\frac{p(D-p)}{D^2}\breve{\rho}$$

For DW p=D-1

Isotropic and Cubic Symmetries

$$\breve{\mu}_{\mathrm{min}} = \frac{\breve{\rho}}{2D}$$

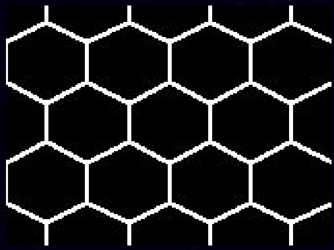
Choose a configuration which verifies a symmetry and you got yourself a RIDE !

What kind of RIDE is safe?

Isotropic Symmetry

2D Solid honeycomb

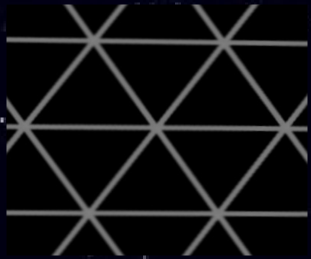
Y type



Unstable with $\mu/\rho = 1/8$

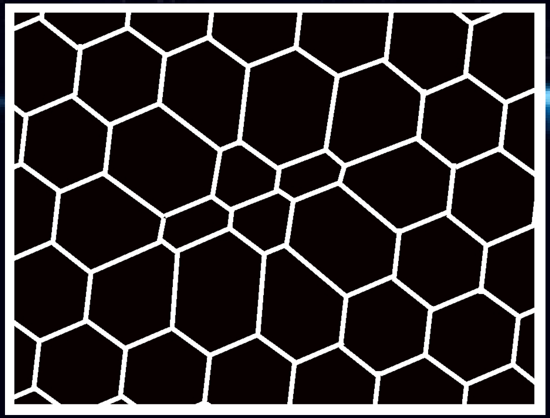
$$\rho = \check{\rho} + \frac{1}{2}\check{\Sigma}^{abcd} s_{ab} s_{cd}$$

2D Solid Triangular



X type

Stable with $\mu/\rho = 3/8$



3D Solid X type Junction

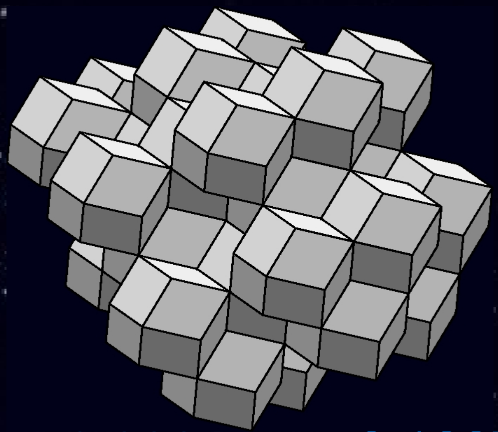
Stable with $\mu/\rho = 4/15$

What kind of RIDE is safe?

3D Anisotropic Examples

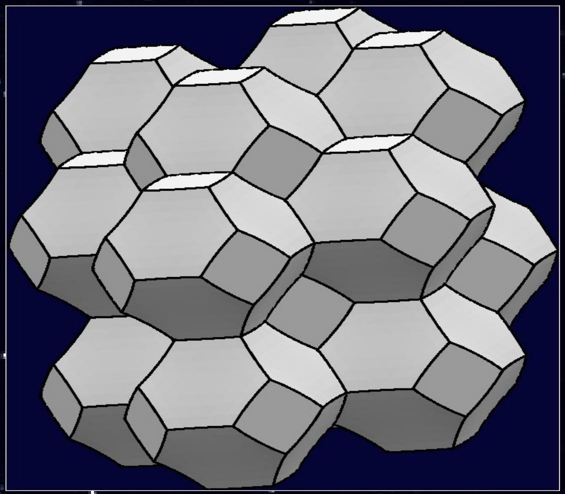
Cubic Symmetry

Rhombic Dodecahedron



μ_L=0.222 μ_T=0.1667

Kelvin Foam (Truncated Ocathedron)



μ_L=0.109 μ_T=0.109

Orthonormal type

$$\mu_L = \rho/2D$$

$$\mu_T = \rho/D$$

Numerical Simulations
Work in Progress..

$$V(\phi_i) = \frac{1}{4}(\phi_i\phi^i - 1)^2 + \epsilon \sum_i \phi_i^4$$

Stability check strategy

Specify an initial lattice

↓

Perturb

↙ ↘

Wiggles   Fluctuations

Vacuum solutions

ε > 0

$$\phi_i = \pm \frac{1}{\sqrt{N + 4\epsilon}}$$

2^N Vaccua

ε < 0

$$\phi_i = \pm \frac{1}{\sqrt{N + 4\epsilon}} \qquad \phi_j = 0 \ \forall i \neq j$$

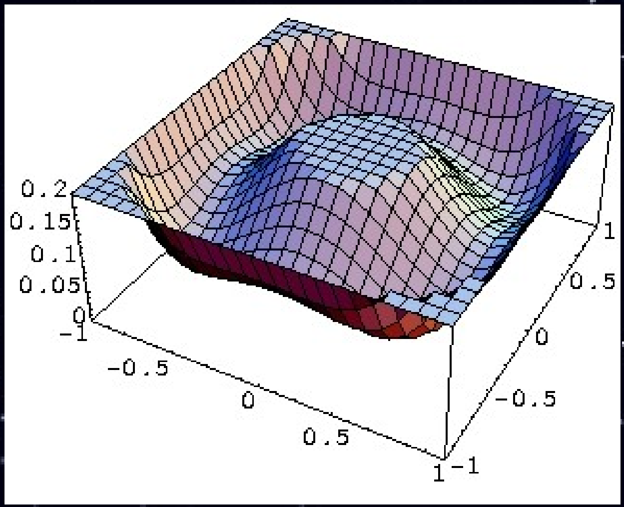
2N Vaccua

First Application of the code
The N=2 case

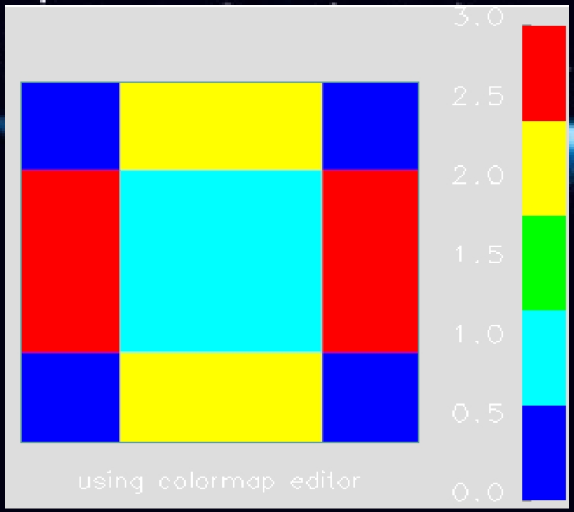
Initial Conditions

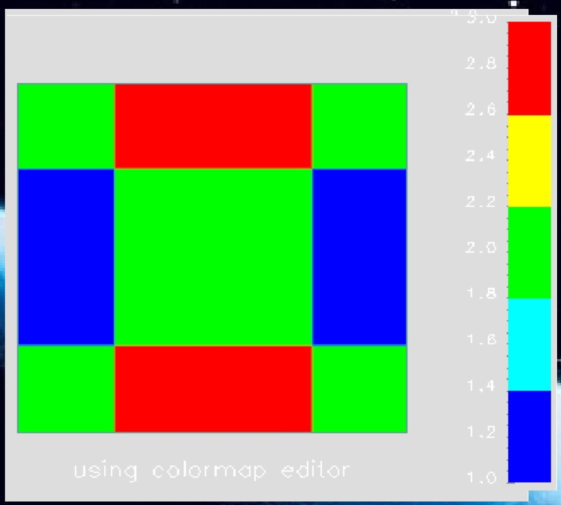
Only two type of frontiers

Regular Wall — Super Wall



Square lattice in 2D





Where do we go from there ?
Conclusions

- **Rigidity Introduced in DE**

  allows for stable models provided sufficient rigidity
  implications in the case of non isotropy

- **Stability analysis**

  Analytical : Orthonormal , Hexagonal in 2D

  Numerical: Kelvin Foam, Rhombic Dodecahedron

- **Numerical Simulations**

  Just Getting Started... more things to come

Thank You